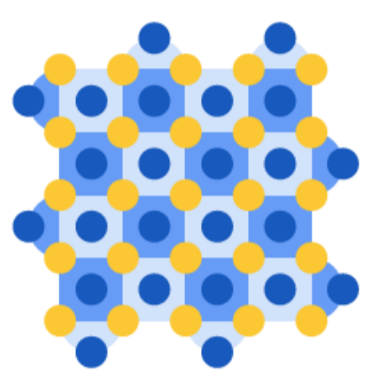


Abstract

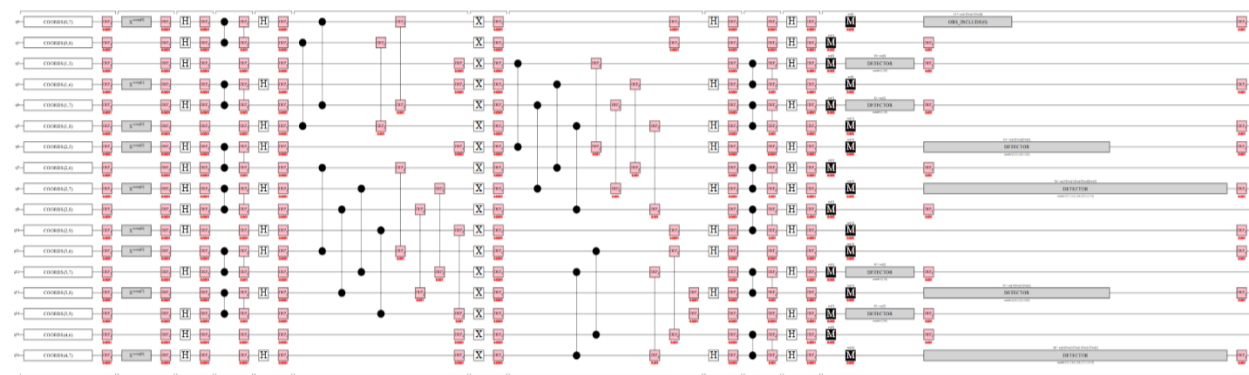
This project provides critical components in overcoming the challenge in quantum error correction, advancing toward real-time decoding error syndromes of a large-scale quantum system at an accelerated speed. We achieve this task by implementing surface code error correction circuits, realizing quantum error correction algorithms, and reproducing the error correction results of the ZXXZ surface code[2] based on a real quantum device. Using the experimental result released by Google Quantum AI Lab, the Willow, containing 107 superconducting qubits[6], we decode its error syndrome of different code distances ($d = 3, 5, 7$) with several decoding algorithms. Additionally, we collaborate with several international partners to implement Helios, a Union Find approach, on FPGA hardware at an elevated decoding speed. We converted the ZXXZ surface code used in the Google Willow chip to a surface code format used in Helios for error correction. We incorporated Stim simulator and implemented three decoding algorithms. Among them, the Union-Find decoder[3, 5] performed the best. On a 72-qubit real-time quantum circuit, its performance can rival to Google's method, while demonstrating superior scalability and speed. In addition, we implemented the Union-Find decoder Helios[8], and successfully verified its functionality on FPGA. Looking forward, we aim to further implement this algorithm on an ASIC to achieve truly real-time quantum error correction.

Introduction

1. Quantum circuit formulation of surface code

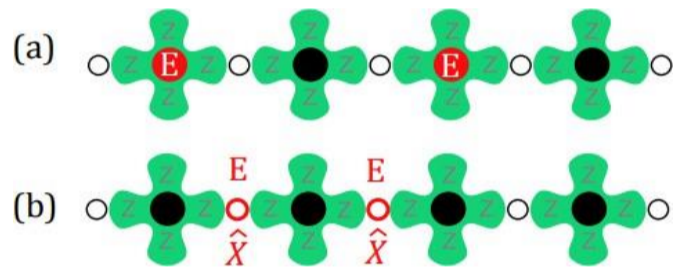


Surface code layout [7]

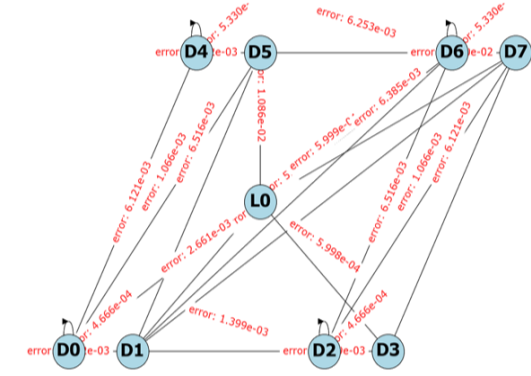


Encoding Circuit for $d=3$ surface code

2. Error pairs, stabilizers and Tanner Graph



Stabilizers & Error pairs [1]



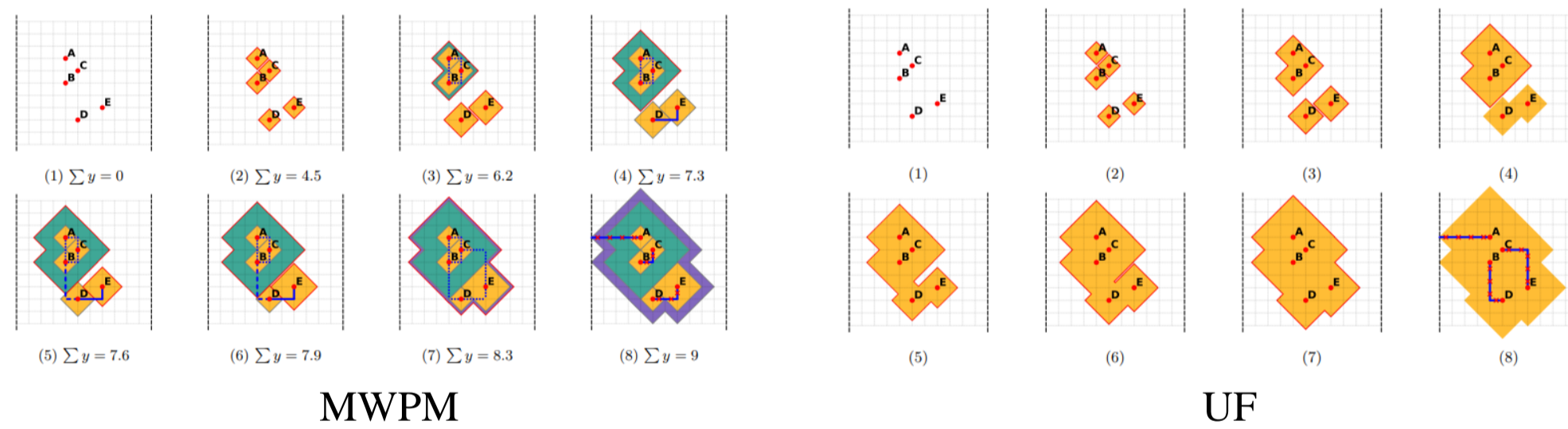
Tanner Graph

Research Method

1. Look Up Table (LUT) & decoding algorithms

Our research emphasizes on graph-based algorithm: MWPM[4] and UF[3, 5] methods. Their time complexity and accuracy are list in the table.

Algorithm	Complexity	Accuracy
LUT	$2^{2^{n/2}}$	-
UF	$O(n \cdot \alpha(n))$	Middle Low
MWPM	$O(n^7)$	Middle



MWPM

UF

2. Decoding algorithm verification on the Willow chip

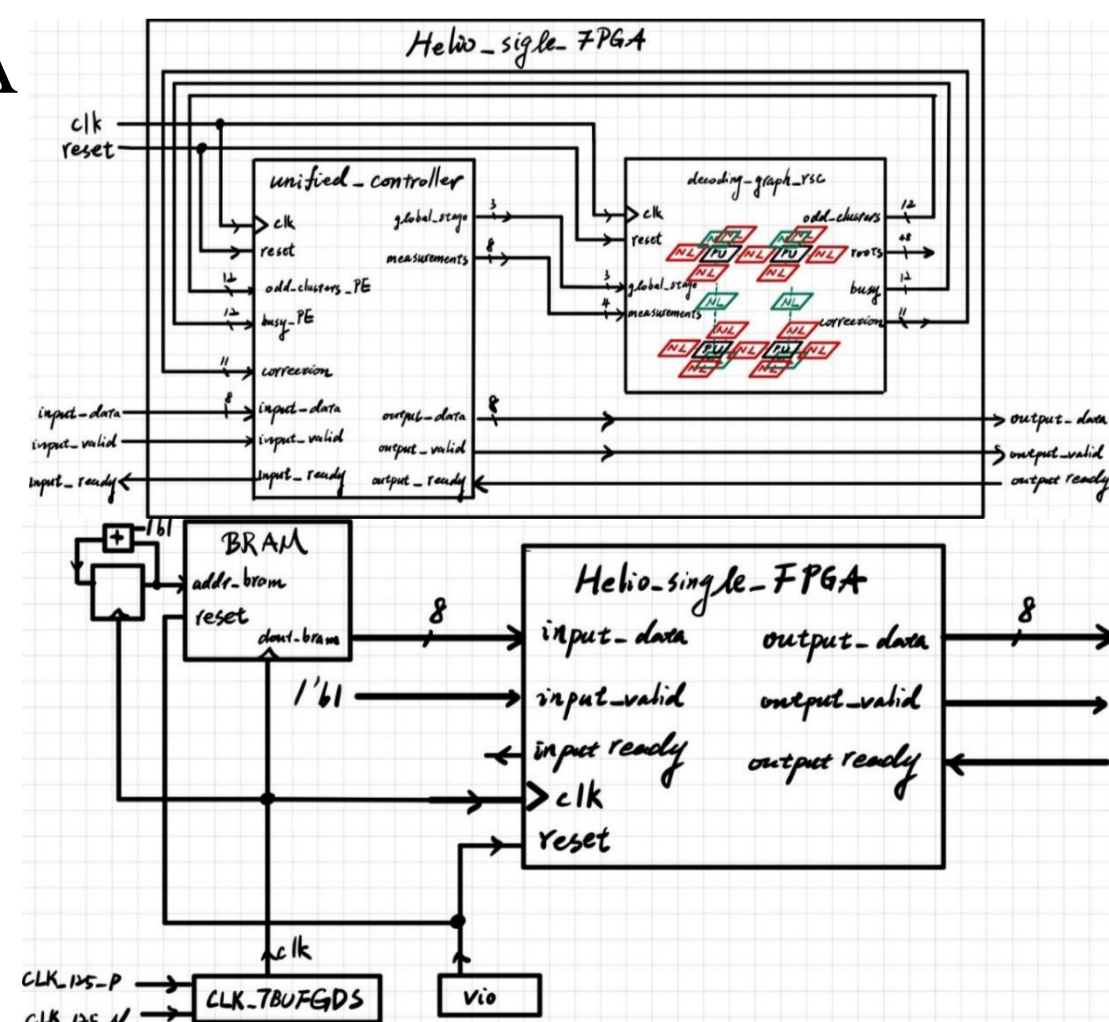
Google Quantum AI team release the QEC result of their Willow [6] chip. The processor consists of 107 superconducting qubits. We simulate its data [7] to verify our approaches. This table shows Willow's dataset information [6].

Dataset	Number of experiments	Basis	Distance	Cycles	Shots
google.72q.surface.code.d3.d5.set1 This surface code dataset corresponds to Fig. 2(e) in the main text, which represents the last 16 experiments that were run consecutively.	21	X, Z	3, 5	1 to 250	5×10^4
google.72q.surface.code.d3.d5.set2 Samples in this surface code dataset capture different calibration states of the device, ranging from well-calibrated with LER of the distance-5 code smaller than 3×10^{-2} to poorly calibrated with LER approaching 10^{-2} . This selection is intended for testing the decoding algorithms under a wide range of experimental conditions.	35	X, Z	3, 5	5 to 50	6×10^4
google.105q.surface.code.d3.d5.d7 This surface code dataset corresponds to Fig. 1(c,d) in the main text.	1	X, Z	3, 5, 7	1 to 250	5×10^4
google.72q.repetition.code.d29 This repetition code dataset corresponds to Fig. 3(a) in the main text.	100	X, Z	29	10^3	10^5

3. Helios[8] (UF) on FPGA

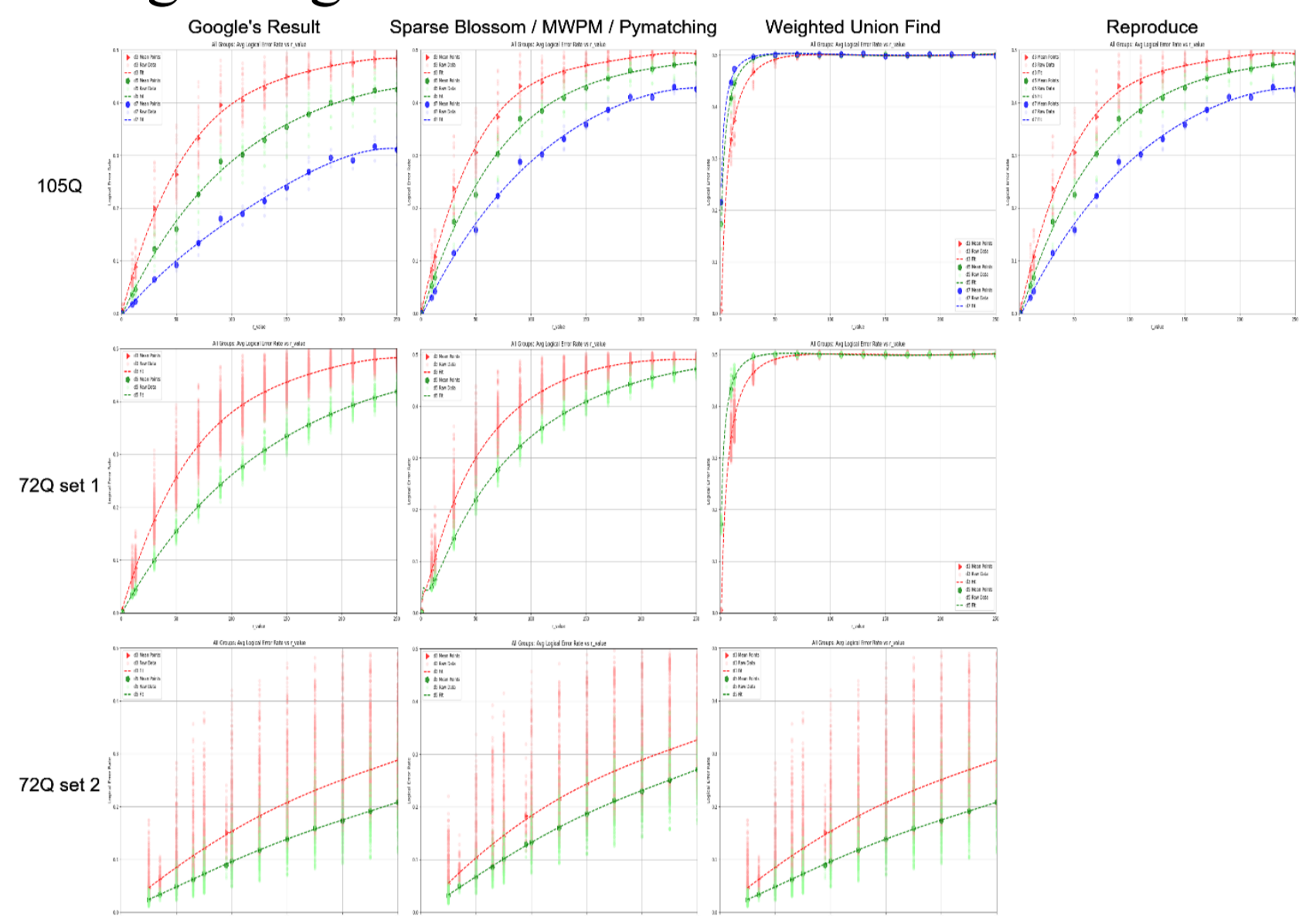
Union-Find (UF) is an efficient algorithm in complexity with lower accuracy compare to the Minimum Weight Perfect Matching (MWPM).

We implement the UF decoder on the FPGA based on Liyanage's work, Helios [8]. This is the architecture of the FPGA system.



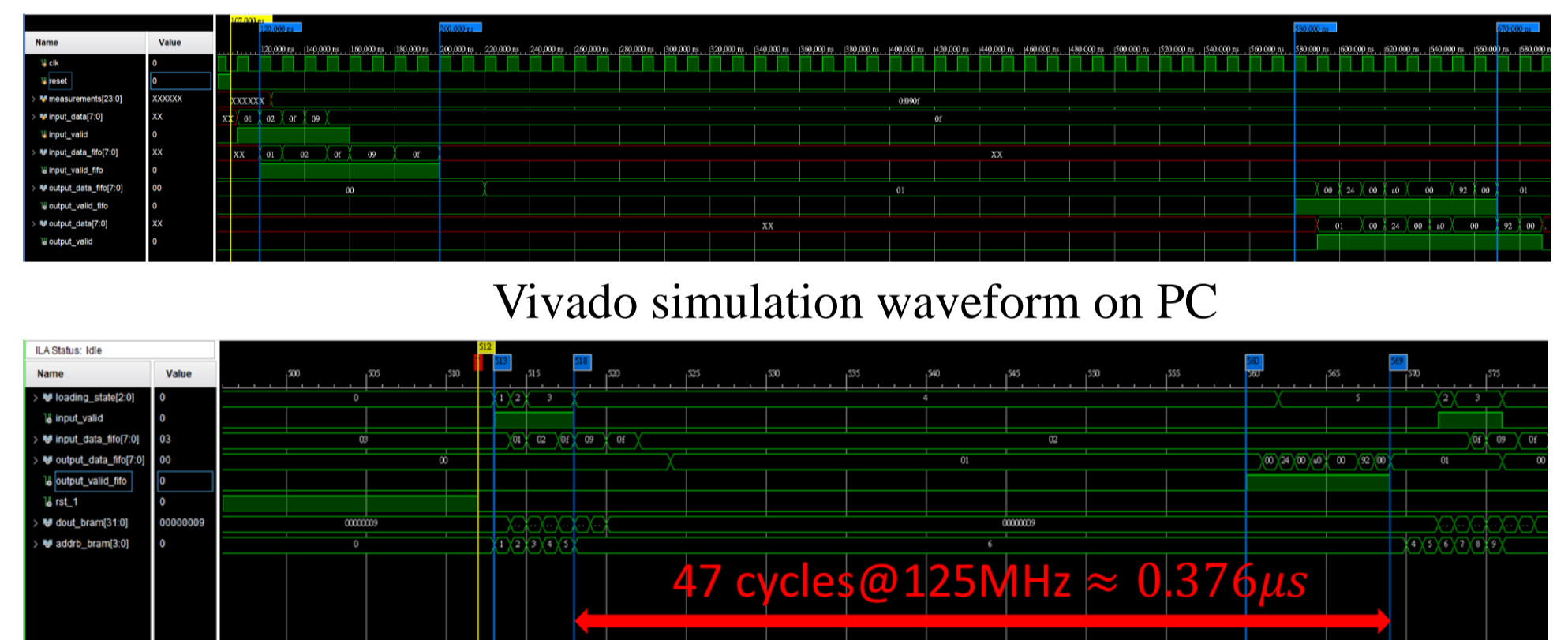
Result

A. Decoding Google's Result



We implemented MWPM and UF algorithms to decode the actual experimental data from the Willow chip. We reproduced Google's results by their published reference [4,6,7] and compare these algorithms. We conclude the 72Q set2 reach below error threshold even with UF decoder's accuracy. In this dataset, Google's team performed additional calibration and lower cycle operation (50 vs 250 cycles).

B. Error Correction of Willow Data on FPGA



Real-time decoding result on FPGA

We converted the syndrome data from Willow's data format and used it as the input to verify the performance of Helios. The real-time decoding on FPGA could be done in $0.376\mu s$, agreeing with the requirement of addressing backlog problem [9].

Conclusion

We implemented the UF algorithm and MWPM to decode syndrome data of a real quantum processor (Google Quantum AI). Under various experimental conditions, UF decoder demonstrates higher decoding rates and lower complexity requirements than MWPM. To implement real-time QEC, it is essential to realize of both algorithms to efficiently decode error syndromes using software and hardware approaches.

In future work, we aim to improve the hardware implementation by upgrading Helios with weighted decoding strategies and implementing the design to ASIC. Such improvements will further reduce decoding latency and enhance the scalability of QEC systems, contributing toward the realization of fault-tolerant quantum computers.

References

- [1] Fowler, Austin G. et al. "Surface codes: Towards practical large-scale quantum computation." *Phys. Rev. A* **86** (2012): 032324.
- [2] Bonilla-Ataides, J. Pablo et al. "The XZZX surface code." *Nat. Comm.* **12** (2020)
- [3] Delfosse, N. and N. H. Nickerson. "Almost-linear time decoding algorithm for topological codes." *Quantum* **5** (2017): 595
- [4] Higgott, Oscar and Craig Gidney. "Sparse Blossom: correcting a million errors per core second with minimum-weight matching." *Quantum* **9** (2025): 1600
- [5] Wu, Yue et al. "An interpretation of Union-Find Decoder on Weighted Graphs." (2022)
- [6] Google Quantum AI. Data for "Quantum error correction below the surface code threshold." *Nature* **638** (2024): 920 - 926
- [7] Acharya, Rajeev et al. "Suppressing quantum errors by scaling a surface code logical qubit." *Nature* **614** (2022): 676 - 681
- [8] Liyanage, Namitha et al. "Scalable Quantum Error Correction for Surface Codes Using FPGA." *2023 IEEE International Conference on Quantum Computing and Engineering (QCE)* 01 (2023): 916-927
- [9] Acharya, Rajeev et al. "Quantum error correction below the surface code threshold." *Nature* **638** (2024): 920 - 926