Discussion on Range of Validity on Applying Tinkham Equation for Conductivity Analysis with THz Spectroscopy

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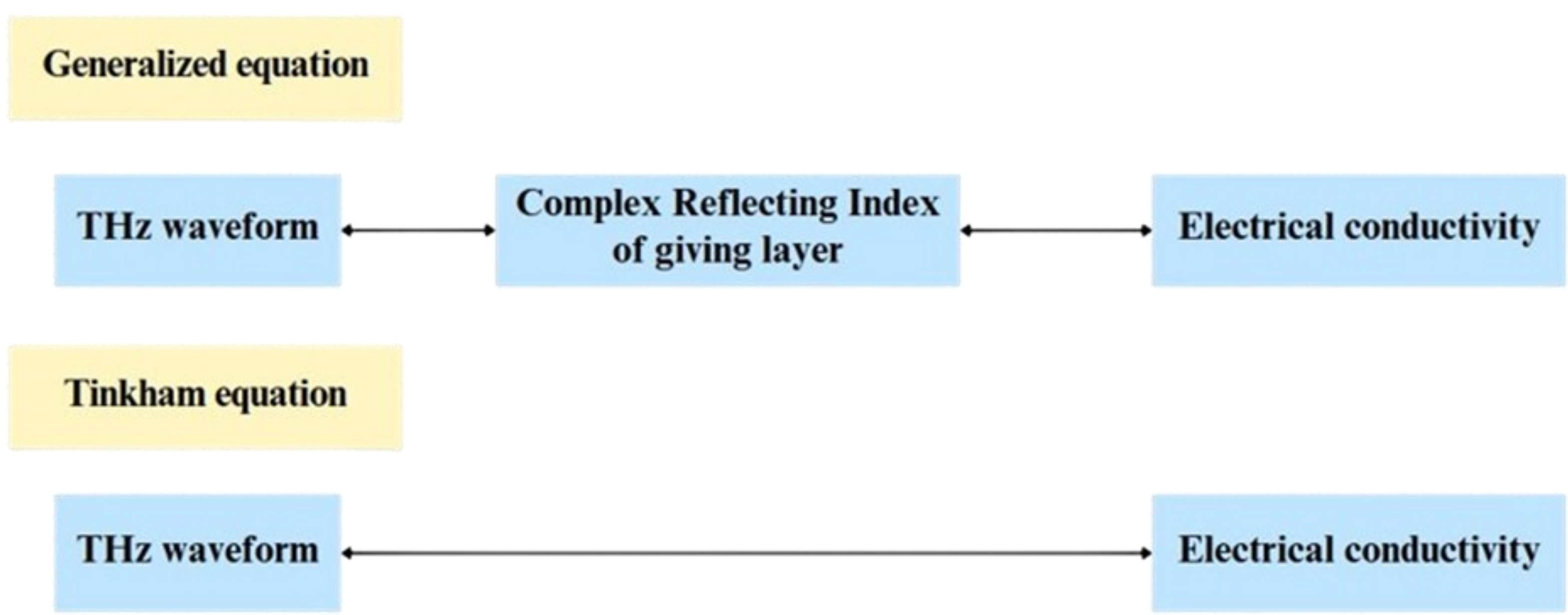
Abstract

This study investigates the applicability of the Tinkham equation in determining the electrical conductivity of thin films via terahertz spectroscopy, using the generalized equation as the theoretical benchmark. Simulations across a range of film thicknesses (1–1000 nm) and conductivities reveal that the Tinkham equation yields accurate results (error <1%) only when the film thickness is significantly less than the terahertz wavelength and skin depth, and the conductivity remains low. Beyond a thickness of approximately 245 nm or at higher conductivities, the error increases markedly, indicating the limitations of the Tinkham equation under these conditions. For instance, in semiconductor applications involving polycrystalline silicon films with typical thicknesses of 30–100 nm and conductivities ranging from 10^2 to 10^4 S/m, the Tinkham equation is applicable primarily within the lower conductivity spectrum (10^2–10^3 S/m). These findings underscore the necessity of pre-assessing material parameters to ensure the validity of the Tinkham equation in terahertz spectroscopic analyses of thin films.

Introduction

In this study, we research the applicability of the Tinkham equation by comparing it with a more rigorous multilayer optical model based on the Fresnel equations and thin-film interference theory, hereafter referred to as the generalized equation. While the generalized equation is theoretically accurate, it lacks a closed-form analytical solution, making it less practical for rapid analysis. Its use typically requires retrieving the complex refractive index of each layer from terahertz (THz) attenuation data and subsequently converting it into electrical conductivity using the Drude model.

By contrast, the Tinkham equation provides a simplified approach: when the film thickness is much smaller than both the THz wavelength and the skin depth, it enables direct estimation of film conductivity from the transmission attenuation of THz waves. This study aims to define the boundary conditions under which the Tinkham approximation remains valid.



Implementation

We implemented a numerical simulation framework using MATLAB to evaluate the error between the Tinkham equation and the generalized equation across a wide range of thin-film parameters. The generalized equation was constructed using Fresnel equations and multilayer interference theory, while conductivity was derived via the Drude model.

We varied the following parameters:

- **Film thickness**: 1 nm to 1000 nm
- **Carrier concentration (Ne)**: 10^{22} to 10^{26} m^{-3}
- **Mobility (μ)**: 0.005 to 0.2 $m^2/V\cdot s$
- **Frequency range**: Terahertz domain (FFT-derived)

For each parameter set, we computed the transmission through the film using both models and defined the **relative error** between them. A threshold of **1% error** was used to determine the validity region of the Tinkham equation.

Using formula:

Generalized equation:

$$\frac{E_{sam}(\omega)}{E_{sub}(\omega)} = \frac{2n_3(n_1+n_2)e^{j(n_3-1)kd_{tf}}}{(n_3+n_2)(n_3+n_1)-(n_3-n_2)(n_3-n_1)e^{j2n_3kd_{tf}}}$$

Tinkham equation:

$$\frac{E_{sam}(\omega)}{E_{sub}(\omega)} = \frac{1+n_{si}}{1+n_{si}+Z_0\sigma(\omega)d}$$

Drude model:

$$\sigma(\omega) = \frac{qNe\mu}{1-i\omega\tau}$$

Result

- (Fig1) Even at the same conductivity, error varies due to Ne–μ asymmetry in refractive index.
- (Fig2) Zigzag patterns arise from multiple error values at the same conductivity due to sampling limitations.
  Increasing parameter resolution improves smoothness and accuracy.
- (Fig3)As film **thickness or conductivity increases**, the **prediction error rises rapidly**, especially beyond a **thickness of ~245 nm**, where even the lowest tested conductivity exceeds the 1% error threshold.
- Simulations show that the **Tinkham equation closely matches the generalized equation** only when the film thickness is **much less than both the THz wavelength and the skin depth**, and when the film conductivity is low.
- **Error maps** plotted across parameter space clearly define the boundary of applicability.

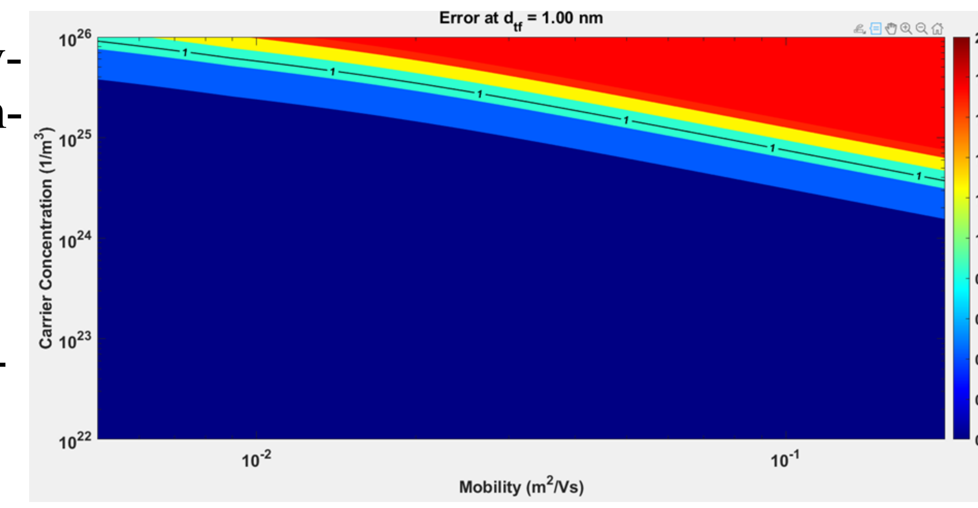


Fig1. Tinkham Equation Error Map at 1 nm Thickness (Ne vs. μ)

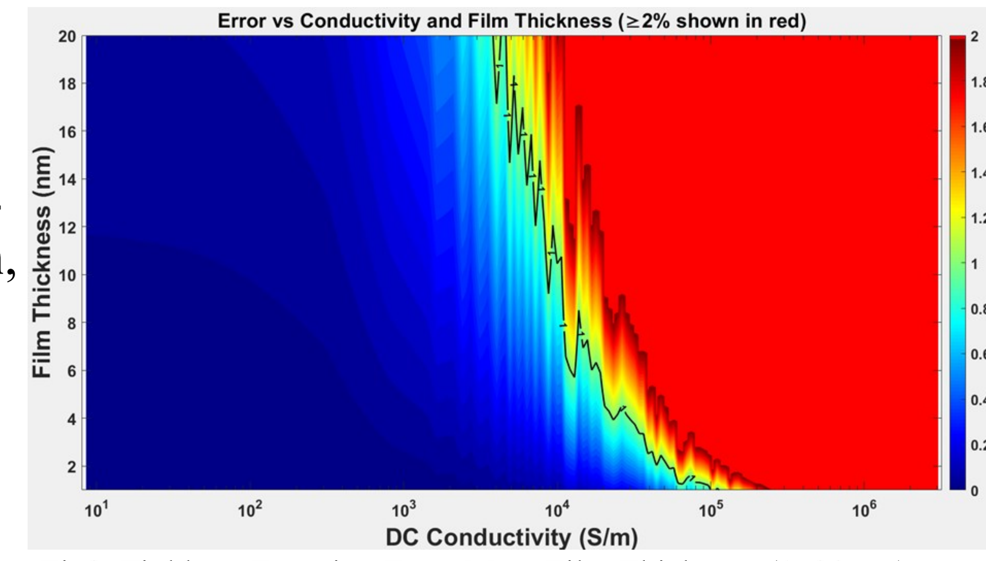


Fig2. Tinkham Equation Error Map: Film Thickness (1–20 nm) vs. Conductivity

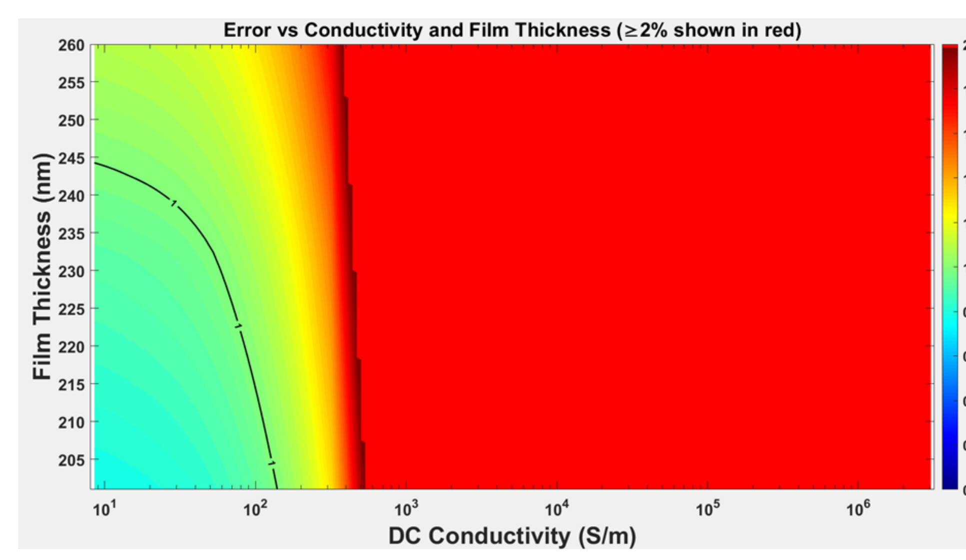


Fig3. Tinkham Equation Error Map: Film Thickness (201–260 nm) vs. Conductivity

Conclusion

This study evaluates the applicability of the Tinkham equation for estimating thin-film conductivity via terahertz spectroscopy, using a multilayer Fresnel-based model as the theoretical reference. Simulation results show that the Tinkham equation is accurate only when the film is significantly thinner than the terahertz wavelength and skin depth, and when the conductivity is relatively low.

As thickness or conductivity increases, the error grows rapidly. Once the film exceeds ~245 nm, even at the lowest simulated conductivity, the Tinkham equation consistently exceeds the 1% error threshold. Moreover, error variation occurs even at constant conductivity due to the asymmetric influence of carrier concentration and mobility on the refractive index. This highlights that conductivity alone does not fully determine the model's accuracy.

We also observed that the discrete nature of simulation sampling introduces zigzag error artifacts, where a single conductivity may correspond to multiple error values. Increasing the resolution of the parameter grid mitigates this issue and improves accuracy.

In summary, the Tinkham equation is a valid approximation only under well-defined physical conditions. This study defines its quantitative limits and demonstrates the need for case-specific validation when applying simplified models in thin-film terahertz analysis.

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