Verifying the feasibility of numerical approximation with Taylor's series on the nonlinear Mach-Zehnder Modulator (MZM) transfer function

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I. Abstract

In this research, the emphasis is on creating and validating an effective model to examine the nonlinear effects of the Mach-Zehnder Modulator (MZM) within the scope of fiber-optic Terahertz (THz) communication systems, a key technology in the transition from 5G to 6G communications. The introduction section underscores the necessity for enhanced data transmission in wireless networks, positioning THz communication as a fundamental component of 6G technology. The paper's primary objective is to provide a comprehensive understanding of the THz wireless channels by developing a mathematical model of the MZM, a critical element in these systems. Additionally, the study assesses the practicality of this model, with a particular focus on its implications for Orthogonal Frequency-Division Multiplexing (OFDM) in upcoming 6G communication networks.

II. Mathematics derivation

The basic transfer function for single-armed MZM in ideal case is

$$E_{out} = \frac{E_{in}}{2}\left(1 + \exp\left(j\frac{r(t) - V_{bias}}{V_\pi}\pi\right)\right) \quad (1)$$

In equation (1), r(t) is the input OFDM signal, V_{bias} is the DC voltage applied on MZM. V_π is the half wave voltage for MZM. E_{out} and E_{in} is the electric field of the output signal and input signal, respectively. However, this is the ideal case. We need to consider some imperfect factors due to manual problems. Since the power of the two lasers in MZM cannot be perfectly equal, we consider two major factors, voltage shift (V_s), and unbalanced power factor (U_b). With the two parameters, the transfer function of the electric field can be written as

$$\frac{E_{out}}{E_{in}} = \frac{1}{2}(1 + U_b + (1 - U_b)\exp\left(j\frac{r(t) - V_{bias} - V_s}{V_\pi}\pi\right) \quad (2)$$

The equation (2) is the working procedure in electric field. Since MZM is the intensity field, where $I_{out} = |E_{out}|^2$. With this relationship between electric field and intensity field, we can get $\frac{I_{out}}{I_{in}} = \frac{|E_{out}|^2}{|E_{in}|^2}$, i.e. $\frac{I_{out}}{I_{in}} = \frac{E_{out}}{E_{in}} \times (\frac{E_{out}}{E_{in}})^*$. After some simple calculations, we can get the following equation.

$$\frac{I_{out}}{I_{in}} = \frac{1}{2}((1 + {U_b}^2) + (1 - {U_b}^2)\left(\cos(\frac{r(t) - V_{bias} - V_s}{V_\pi}\pi)\right)) \quad (3)$$

To analyze the nonlinear effect of the transfer function, we do a Taylor's expansion on the cosine function to get equation (4), which contains a linear term (first order term) and nonlinear terms (higher order terms). Note that in equation (4), $V_{ope} = V_{bias} + V_s$.

$$\frac{I_{out}}{I_{in}} = \frac{1}{2}\{(1 + {U_b}^2) + (1 - {U_b}^2)\sum_{n=0}^{\infty}[\frac{(-1)^n}{(2n)!}(\frac{r(t) - V_{ope}}{V_\pi}\pi)^{2n}]\} \quad (4)$$

III. Simulation results

From equation (4), Taylor's series can be separated into linear terms (first order term) and nonlinear terms (higher order terms) in the MZM transfer function. To discuss the nonlinear effect of MZM, we sent sine waves with different peak-to-peak voltages (V_{pp}) to test the performance of different orders of nonlinear terms using MATLAB simulation. From the result, we obtain a graph in Figure 1.

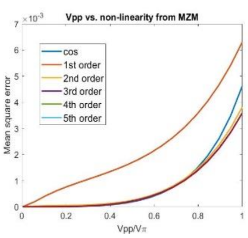

Fig. 1 The nonlinearity problem on MZM with different order of Taylor's series and different Vpp

We can easily see that the performance of those terms with order higher than two are very similar, which means we can only use the first two terms to approximate the MZM transfer function to get a good trade-off between calculating complexity and accuracy. Thus, we can get equation (5)

$$\frac{I_{out}}{I_{in}} = \frac{1}{2}\{(1 + {U_b}^2) + (1 - {U_b}^2)\sum_{n=0}^{2}[\frac{(-1)^n}{(2n)!}(\frac{r(t) - V_{ope}}{V_\pi}\pi)^{2n}]\} \quad (5)$$

To verify the feasibility of this simplified version MZM transfer function, we use MATLAB to simulate two different transfer functions (the simplified version and the realistic function). In this simulation, we transmit an OFDM signal. The figure below is the input signal when entering MZM.

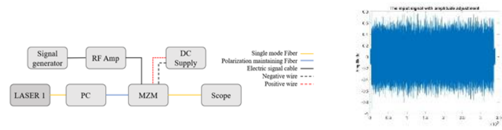

Fig. 2 (a) The block diagram of the fiber-optic communication system we use in simulation
(b) The waveform of input OFDM signal with real case amplitude before entering MZM

In the following outcome, the peak-to-peak voltage of OFDM signal, $V_{pp} = 0.8\,Volt$, and for the realistic case of MZM, we set $V_\pi = 3\,Volt$.

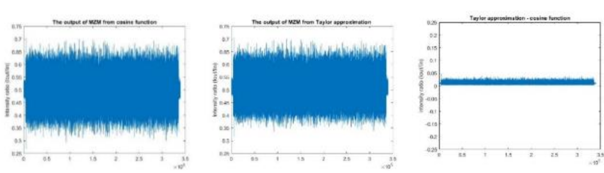

Fig. 3 The intensity ratio of (a) the output of real case MZM transfer function
(b) the output of the Taylor's approximation
(c) real case function minus Taylor approximation function

For Fig. 3(a) and Fig. 3(b), we found that the intensity ratio ($\frac{I_{out}}{I_{in}}$) of MZM from the actual case and the Taylor approximation one is almost identical. (Notice that we depict those terms with order higher than two, and the summation of them are negative. Thus, the waveform of the Taylor approximation will be shifted upward by a small value.)

To verify the similarity, we subtract the output waveform of the real case from the Taylor approximation transfer function to get Fig. 3(c). The standard deviation of the **subtracted waveform** is **0.0039**, and the standard deviation of **real case** and **Taylor approximation** are **0.0488** and **0.0450**, respectively. The difference between two waveform is less than 10%, which is a negligible error.

IV. Conclusion

In the numerical simulation, we verify that we can use only the first two terms of the Taylor series to approximate the MZM transfer function, which decreases the calculating complexity while maintaining accuracy within a reasonable scope. The results give us a better way to discuss the nonlinear effect of the transfer function of MZM. Also, it helps us predict and calculate the performance of fiber-optic THz communication systems more efficiently, contributing to the development of sixth-generation wireless communication technology.