國立清華大學 電機工程學系

實作專題研究成果摘要

A New Upper Bound on the Buffer Sizes of SDL Constructions of Optical Priority Queues

SDL 構造光優先佇列容量的新理論上界

專題領域：系統領域

組　　別：A511

指導教授：鄭傑

組員姓名：曾學文　王瑋鑫

研究期間：2024年7月1日至 2025年5月1日止，共 10 個月

Abstract

As the Internet becomes increasingly important and wide-spread nowadays, some people start trying to build buffers with optical fibers and optical switches, which can avoid O-E-O (optical-electrical-optical) conversion in networks. These constructions give a brand-new idea of making memories in a pure optical network. However, unlike electronic memories, optical memories face a big challenge that data moves around in the systems, and it turns out to be impossible to make a memory by simply designing the buffer as electronic ones.

To conquer the difficulty, researchers came up with an idea of building a buffer using SDL (Switch and Delay Line) constructions. Furthermore, a feedback system architecture constructed by M delay lines and a (M+1)X(M+1) optical switch is proposed and accepted by researchers in this field.

After the most common-seen FIFO queue is constructed, researchers began to design priority queues in this scheme. Currently, the best SDL construction using feedback architecture with M delay lines and a (M+1)X(M+1) optical switch reaches a maximum buffer size of order O($2^{\sqrt{M}}$). While the best-known result in this field gave an upper bound of $2^M - 1$.

This work is basically interested in finding an upper bound of the SDL construction in feedback architecture, known as the most applicable structure of optical buffers, of an optical priority queue. In our work, we proved that the system has an upper bound of 2^{M-1} with some recursive restriction on the length of every delay line in the system.

Introduction

1. Research Background

As we mentioned previously, this report focuses on the proof of the new upper bound of SDL construction under feedback architecture. We discovered that the recent upper bound is way too far from the construction buffer size, and thus we wonder if the upper bound was too loose. We then read papers and work on a mathematical proof to lower the upper bound.

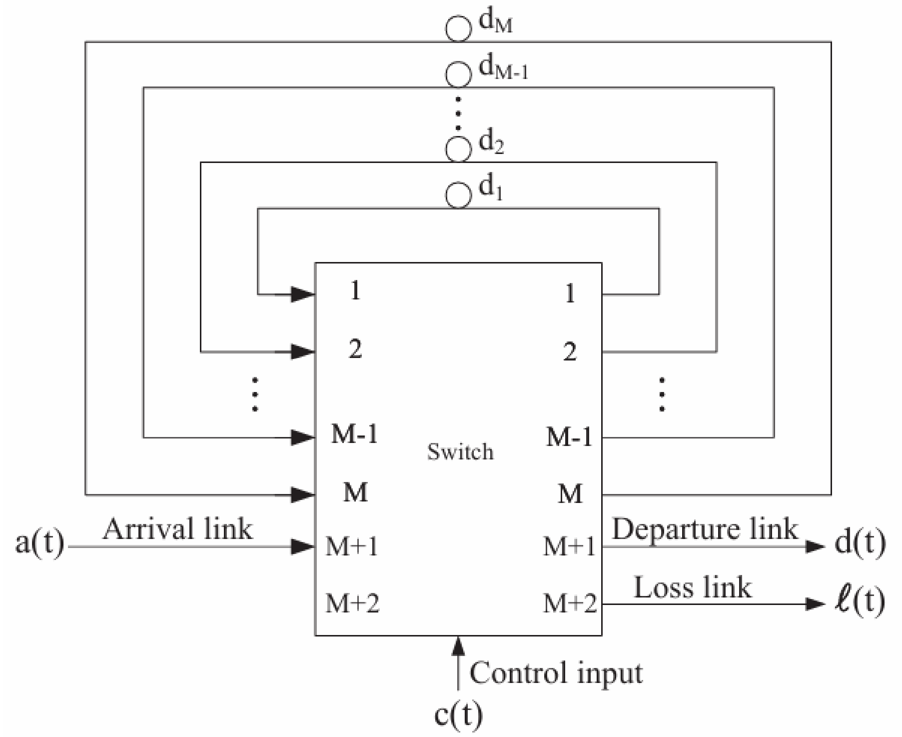

Fig. 1 A feedback architecture for the implementation of an optical priority queue (PQ)

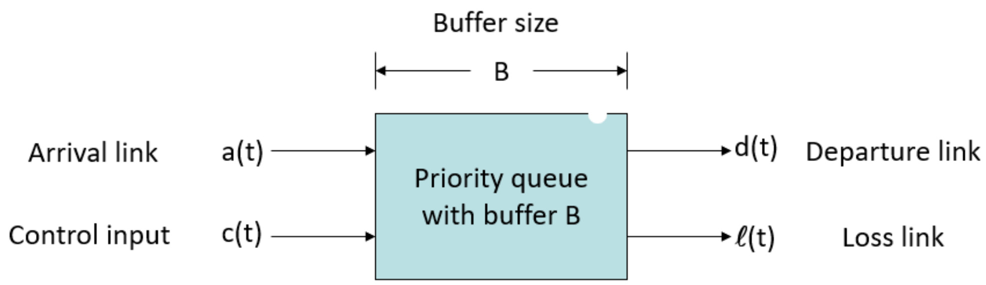

Fig. 2 A priority queue (PQ) with buffer size B is a network element with one control input, one arrival link, one departure link, and one loss link.

2. Research Purpose

The purpose of this work is to showcase the new upper bound we found. In other words, we proved that any optical priority queue in SDL constructions using feedback system architecture cannot have a buffer size greater than 2^{M-1}. Meanwhile, we also give a proof that every used delay line (in some cases, that packets may be routed into the specific delay line) has a length upper bound, which actually leads to the result of the new upper bound since the buffer size is the maximum number of packets that can be stored in the delay lines, and therefore the buffer size is restricted by the length of delay lines.

3. Research Method

We give the definitions of priority queue and buffering tags first, then explain how we complete our proof.

Definition of priority queue:
Priority queue with buffer size B is characterized by the following five properties:

(P1) Flow conservation: Packets arriving from the arrival link are either stored in the buffer or transmitted through the departure link or the loss link. Thus, we have $q(t)=q(t-1)+a(t)-d(t)-l(t)$.

(P2) Nonidling: There is a departure packet at slot t only when there is a departure request from the controller and there are packets in the queue at slot t. Thus, we have $d(t)=1$ if $c(t)=1$ and $q(t-1)+a(t) > 0$, and $d(t)=0$ otherwise.

(P3) Maximum buffer usage: There is a loss packet at slot t only when there is a buffer overflow at slot t. Thus, we have $l(t)=1$ if $c(t)=0$, $q(t-1) = B$, and $a(t)=1$, and $l(t)=0$ otherwise.

(P4) Priority departure: If there is a departure packet, say packet p, at slot t, then packet p is the packet with the highest priority in the queue at slot t, i.e., $\tau_p(t)=1$.

(P5) Priority loss: If there is a loss packet, say packet p, at slot t, then packet p is the packet with the lowest priority in the queue at slot t, i.e., $\tau_p(t)=B+1$.

Definition of buffering tags:
the priority level of the packets that have to be stored in the buffers. For example, ith-highest-priority packet among all of the packets that have
to be buffered in the PQ equal to i.

Our proof starts by showing that packets cannot be routed into delay lines that are too short, otherwise the packet may not reach the switch in time, resulting in a construction failure. After we obtain this result, we see that if the number of delay lines that are shorter than a specific number is too few, then the system will fail due to lack of choices of delay lines. Then we introduced an algorithm to classify delay lines into three groups and came up with a result that for the first two groups in the system, a specific bound is necessary, while all the delay lines in the third group will not be used in the system. Finally, this bound leads to the conclusion that the bound of the buffer size is 2^{M-1}.

4. Experimental Results

There are five main results in our work.

1. Every packet at the input of the switch that has buffering tag "p" and complementary buffering tag "q" cannot be routed into any delay line that is longer than min(p,q)

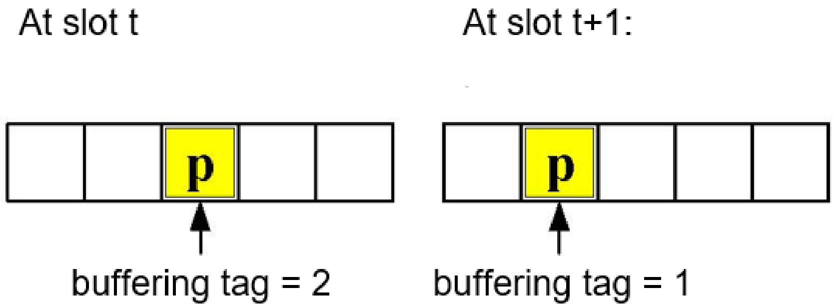

Fig. 3 A visualization of our first result

We see that the packet with buffering tag 1 doesn't reach the last cell of the delay line, thus when a departure request is claim at the next time slot, the packet will not be able to depart from the system, and ends in a failure of construction.

2. If the buffer size of the system is greater than 1 (B>1), then there's at least two delay lines with length 1.
3. Any optical priority queue under feedback system architecture that satisfies the following two statements,

C(1) : $d'_{j+1} \leq \sum_{i=1}^{j} d'_i + 1$ for $1 \leq j \leq n_1 - 1$

and $d''_{j+1} \leq \sum_{i=1}^{j} d''_i + 1$ for $1 \leq j \leq n_2 - 1$

C(2) : if $n_1 + n_2 < M$, then $d_{n_1+n_2+1} > \max\{\sum_{i=1}^{j} d'_i + 1, \sum_{i=1}^{j} d''_i + 1\}$

it will satisfy :

$$B \leq \sum_{i=1}^{n_1} d_i' + \sum_{i=1}^{n_2} d_i'' \leq 2^{n_1} + 2^{n_2} - 2$$

$\{d_1', d_2', \dots, d_{n_1}'\}$ and $\{d_1'', d_2'', \dots, d_{n_2}''\}$ are distinct delay lines in the system

4. There is an algorithm that can group delay lines in any system under feedback architecture so that the groups of the delay line will satisfy C1 and C2 in 3.
5. Any optical priority queue under feedback architecture with M delay lines and buffer size B satisfies $B \leq 2^{M-1}$

5. Conclusion

Compared to previous results from other researches, which primarily focused on construction, we introduced a novel perspective to discuss the upper bound of an optical priority queue under feedback system architecture. We not only proved that there's a tighter bound, but also constructed a recursive bound on every delay line in the system. This work might be extended in the future and the upper bound may become lower and lower until it meets the construction buffer size.

6. Reference

[1] A. D. Sarwate and V. Anantharam, "Exact emulation of a priority queue with a switch and delay lines," Queueing Syst., Theory Appl., vol. 53, no. 3, pp. 115–125, Jul. 2006.

[2] H.-C. Chiu, C.-S. Chang, J. Cheng, and D.-S. Lee, "A simple proof for the constructions of optical priority queues," Queueing Systems: Theory and Applications, to be published.

[3] J. Cheng, S.-H. Yang, C.-Y. Wang, H.-H. Tang, and B. Tang, "On efficient constructions of optical priority queues," IEEE Transactions on Communications, vol. 70, no. 3, pp. 1861 1874, March 2022. A full version is available online at https://www.ee.nthu.edu.tw/jcheng/publications/PQ-efficient constructions-full-version.pdf.

[4] J. Cheng, H.-H. Chou, L.-C. Chang, S.-S. Huang, H.-W. Tseng, and C.-H. Yang, "On constructions of optical priority queues under a priority-based routing policy," submitted to IEEE Transactions on Information Theory, August 2023. Available online at https://www.ee.nthu.edu.tw/jcheng/publications/PQ-PBR full-version.pdf.

7. Review and reflections

學文：

It was tough work to formalize the proof of the thought that, in general, packets are not able to be routed to delay lines that are too long for them. Beyond that, the general case which focuses on the whole system is way more challenging and it was really satisfying when we finished our work. Working on proofs is tiring, though, I learned a lot and I'm looking forward to the next job to be finished in this field.

瑋鑫：

In the process of topic study, I read several papers about SDL constructions. I learned that we could build FIFO optical mux and FIFO optical queue with SDL structure, and I found it very interesting. Finally, we worked on the proof of the new upper bound on the buffer sizes of SDL optical priority queues, the proof was hard, and it took a lot of time to finish our work. I learned a lot and I am very accomplished for what we have done in these months. I am grateful to our professor and lab seniors for all the guidance and support they provided.