SDL構造光優先佇列容量的新理論上界

A New Upper Bound on the Buffer Sizes of SDL Constructions of Optical Priority Queues

組別：A511　組員：曾學文、王瑋鑫　指導教授：鄭傑

國立清華大學
NATIONAL TSING HUA UNIVERSITY

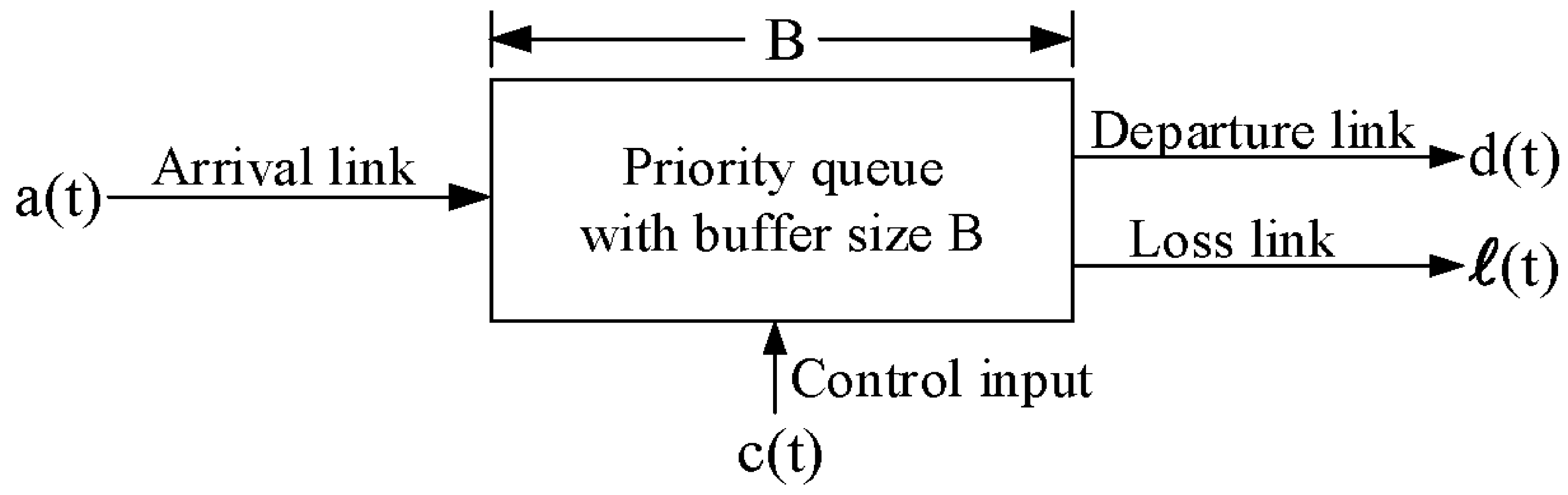
Introduction

As the Internet becomes increasingly important and wide-spread nowadays, some people start trying building buffers with optical fibers and optical switches, which can avoid O-E-O (optical- electrical-optical) conversion in networks. However, unlike electronic memories, optical memories faces a big challenge that data moves around in the systems and it comes out to be impossible to make a memory by simply design the buffer as electronic ones. To conquer the difficulty, researchers came up with an idea of building a buffer using SDL (Switch and Delay Line) constructions under feedback system architecture.

This work is basically interested in finding an upper bound of the SDL construction in feedback architecture, known as the most applicable structure of optical buffers, of an optical priority queue.

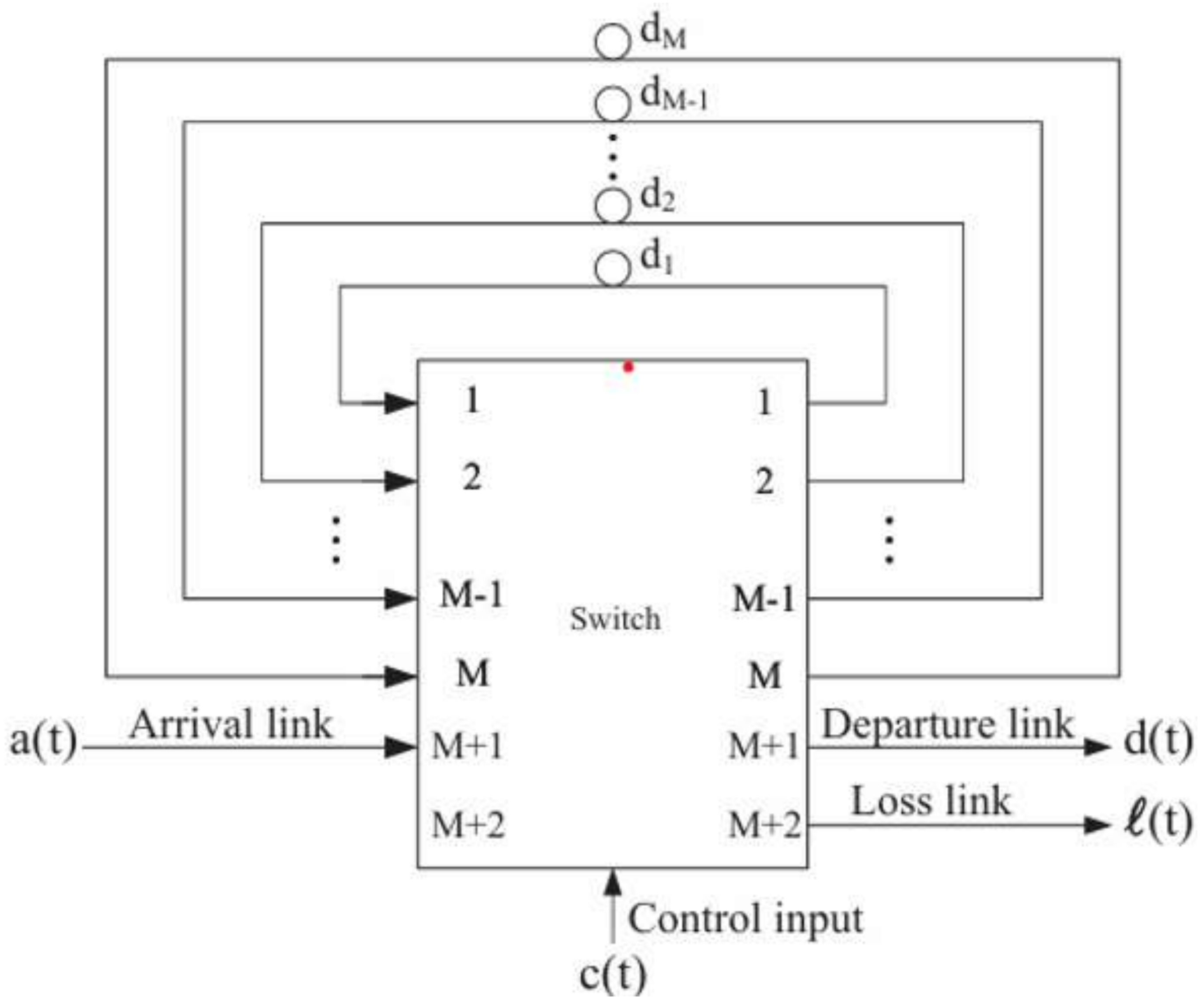
In our work, we proved that the system has an upper bound of 2^{M-1} with some recursive restriction on the length of every delay line in the system.

A Definition of a Priority Queue (PQ)



(P1) Flow conservation: Packets arriving from the arrival link are either stored in the buffer or transmitted through the departure link or the loss link. Thus, we have q(t)=q(t−1)+a(t)−d(t)−l(t).
(P2) Nonidling: There is a departure packet at slot t only when there is a departure request from the controller and there are packets in the queue at slot t. Thus, we have d(t)=1 if c(t)=1 and q(t − 1) +a(t) > 0, and d(t)=0, otherwise.
(P3) Maximum buffer usage: There is a loss packet at slot t only when there is a buffer overflow at slot t. Thus, we have l(t)=1if c(t)=0, q(t − 1) = B, and a(t)=1, and l(t)=0 otherwise.
(P4) Priority departure: If there is a departure packet, say packet p, at slot t, then packet p is the packet with the highest priority in the queue at slot t, i.e., $\tau_p(t)=1$.
(P5) Priority loss: If there is a loss packet, say packet p, at slot t, then packet p is the packet with the lowest priority in the queue at slot t, i.e., $\tau_p(t)=B+1$

A Feedback System Architecture



Main Results

1. Every packet at the input of the switch that has buffering tag "p" and complementary buffering tag "q" cannot be routed into any delay line that is longer than min(p,q).

Explanation:
By contradiction, if a packet "P" with buffering tag entered a delay line with length 4, then after two time slot in which a packet is sent out of the system while no packet enter the system, packet "P" will has buffering tag 1 while it still need an extra time slot to reach the input of the switch, which means that it cannot be sent out of the system and causes a failure on construction.

2. If the buffer size of the system is greater than 1 (B>1), then there's at least two delay lines with length 1.

3. Any optical priority queue under feedback system architecture that satisfies the following two statements,

C(1): $d'_{j+1} \le \sum_{i=1}^{j} d'_i + 1$ for $1 \le j \le n_1 - 1$
And $d''_{j+1} \le \sum_{i=1}^{j} d''_i + 1$ for $1 \le j \le n_2 - 1$

C(2) : if $n_1 + n_2 < M$,
then $d_{n_1+n_2+1} > \max\{\sum_{i=1}^{j} d'_i + 1,\ \sum_{i=1}^{j} d''_i + 1\}$

it will satisfy:

$$B \le \sum_{i=1}^{n_1} d'_i + \sum_{i=1}^{n_2} d''_i \le 2^{n_1} + 2^{n_2} - 2$$

$\{d'_1, d'_2, \dots, d'_{n_1}\}$ and $\{d''_1, d''_2, \dots, d''_{n_2}\}$ are distinct delay lines in the system.

4. There is an algorithm that can group delay lines in any system under feedback architecture so that the groups of the delay line will satisfy C1 and C2 in 3.

5. Any optical priority queue under feedback architecture with M delay lines and buffer size B satisfies $B \le 2^{M-1}$

Explanation:
By 3., we see that the maximum case for an optical priority queue with M delay lines and an (M+1)X(M+1) optical switch is to assign the length of the delay lines by 1,1,2,4,8,16,32,... which ends up having a total buffer size 2^{M-1}

Conclusion

Compared to previous results from other researchers, which basically worked on construction of priority queues, we gave a brand-new thought to discuss the upper bound of an optical priority queue under feedback system architecture. We not only proved that there's a tighter bound but also constructed a recursive bound on every delay line in the system. This work might be extended in the future and the upper bound may become lower and lower until it meets the construction buffer size.

Reference

[1] A. D. Sarwate and V. Anantharam, "Exact emulation of a priority queue with a switch and delay lines," Queueing Syst., Theory Appl., vol. 53, no. 3, pp. 115–125, Jul. 2006.
[2] H.-C. Chiu, C.-S. Chang, J. Cheng, and D.-S. Lee, "A simple proof for the constructions of optical priority queues," Queueing Systems: Theory and Applications, to be published.
[3] J. Cheng, S.-H. Yang, C.-Y. Wang, H.-H. Tang, and B. Tang, "On efficient constructions of optical priority queues," IEEE Transactions on Communications, vol. 70, no. 3, pp. 1861 1874, March 2022. A full version is available online at https://www.ee.nthu.edu.tw/jcheng/publications/PQ-efficient constructions-full-version.pdf.
[4] J. Cheng, H.-H. Chou, L.-C. Chang, S.-S. Huang, H.-W. Tseng, and C.-H. Yang, "On constructions of optical priority queues under a priority-based routing policy," submitted to IEEE Transactions on Information Theory, August 2023. Available online at https://www.ee.nthu.edu.tw/jcheng/publications/PQ-PBR full-version.pdf.