

Optimization and Analysis of Schedule Diversity Decoding for Polar Codes on LDPC-Like Sparse Graphs

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Introduction

This study investigates scheduling diversity strategies for Polar Codes under Residual Belief Propagation (RBP) decoding. Pruning techniques are first applied to convert the original Polar factor graph into an LDPC-like sparse graph, reducing complexity and improving RBP efficiency. Node-Wise RBP (NW-RBP) further mitigates greedy behavior in traditional RBP, and the Modified NW-RBP combined with Schedule Diversity enhances variability and improves error-correction performance on systematic Polar Codes (1024, 512). To address greedy group issues and error-floor effects at medium to high SNR, this study introduces Quota-based NW-RBP (Q-NW-RBP), which limits the number of check-node updates per decoding attempt. This prevents excessive updates on a small subset of nodes and effectively lowers the BER floor. Overall, the proposed strategies provide practical improvements for Polar Code decoders.

Fundamentals

• Sparse Graph Pruning

Pruning transforms a polar code into a lower-complexity LDPC-like code by removing Frozen variable nodes (VNs) and redundant check nodes (CNs). The pruning procedure typically includes the following steps:

1. Removal of Frozen VNs
2. Removal of Degree-1 CNs
3. Removal of Condensed VN-CH and VN-Hidden

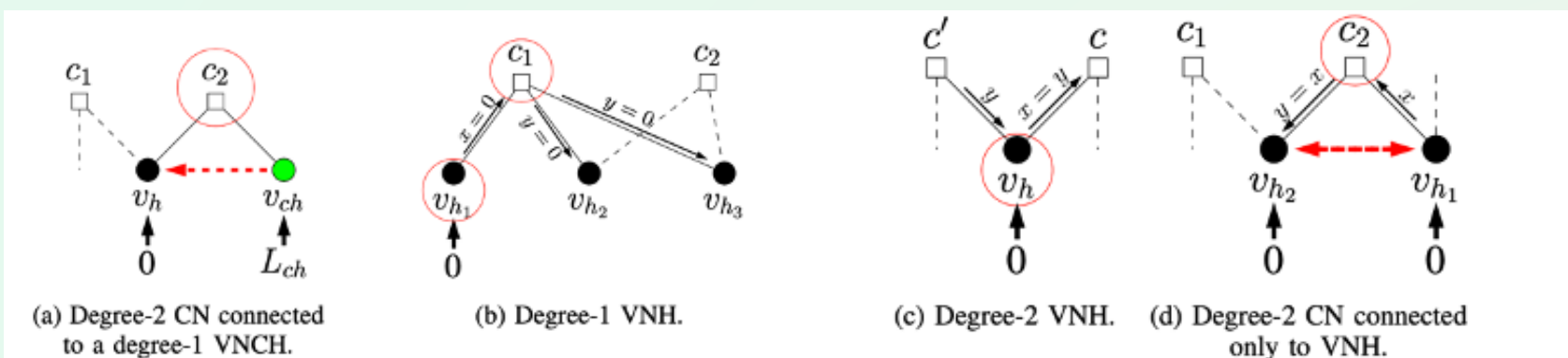


Fig. 1 Special cases of polar factor graph nodes.

Methods

• Layered BP, LBP

It is a kind of BP decoding in which the check nodes (CNs) are divided into multiple layers. In each iteration, only one layer is updated, and the messages are immediately passed to the VNs, enabling faster information propagation. Compared with flooding BP, LBP achieves faster convergence and lower error rates for the same number of iterations.

• Modified Node-Wise RBP (Modified NWRBP)

NW-RBP generalizes traditional RBP by switching the update unit from individual edges to entire nodes. In the modified version, one node is randomly selected from the top- α residuals ($\alpha = 5$ in this research), and all edges connected to that node are updated simultaneously. This prevents any single path from dominating and enhances message-passing diversity.

• Schedule Diversity

Schedule diversity means decoding the same received message using different update schedules and then selecting the most reliable result. In this research, we adopt one iteration of LBP followed by nine iterations of Modified NW-RBP. If LBP fails, the decoder continues with Modified NW-RBP. This approach increases the diversity of decoding attempts, reduces the chance of converging to an incorrect codeword, and preserves the soft-in/soft-out characteristics of BP decoding.

Algorithm : Modified NW-RBP with Schedule Diversity

- 1: Save all L_v^{ini} values
- 2: Set the flag PC = false.
- 3: Set decoding attempt number $i = 1$ and assume the maximum decoding attempt number to be I_{max}
- 4: Perform the LBP considering all L_v^{ini} values. $i = i + 1$
- 5: If all the CNs are satisfied, set PC = true
- 6: While $i \leq I_{max}$ and PC = false
Perform Modified NW-RBP considering all L_v^{ini} values. However, α is randomly determined from $\mathcal{A} = \{1, 2, \dots, \Gamma\}$ in each update. If more than one edge that has the first Γ largest residual value locates in the same CN, increase the size of \mathcal{A} such that we always have Γ CNs to choose in an update.
 $i = i + 1$.
If all the CNs are satisfied, set PC = true
- 7: End While
- 8: Return the decoded results in the last decoder.

Fig. 2 Algorithm of Modified NW-RBP with Schedule Diversity

Reference

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Result

• Optimization algorithm

Edges or nodes with persistently large residuals tend to form a "greedy group," causing updates to concentrate on the same small set of nodes. Even with schedule diversity, Modified NW-RBP remains vulnerable to short cycles that repeatedly reinforce residuals locally, leading to the same group of CNs being selected and amplifying local biases. Therefore, we propose:

Q-NW-RBP

QRBP limits the number of updates for each message on an edge, which effectively mitigates the greedy-group problem. We extend this idea to Modified NW-RBP by restricting the number of updates each CN is allowed to perform within a single decoding process, where λ denotes the maximum number of updates. Simulation results show that setting $\lambda = 150$ yields performance superior to the original algorithm.

• Result analysis and comparison

Applying the optimal λ to Q-NW-RBP with Schedule Diversity yields the results shown in Fig. 4. At low SNR, the initial LLRs are highly unreliable and most CN residuals are similar, making λ ineffective for distinguishing update priorities. Therefore, no clear performance gain is observed in Fig. 4. In contrast, the BER improves significantly in the high-SNR region. This is because the channel LLRs are more reliable, and the performance is mainly limited by the overcorrection and greedy-group behavior of Modified NW-RBP. CNs with larger residuals tend to be repeatedly updated due to short-cycle reinforcement, causing excessive LLR amplification, local instability, and an elevated error floor. Introducing λ limits the frequency of CN updates, distributes update opportunities more evenly, prevents local message amplification and resource concentration, and thus enhances decoding performance in the high-SNR regime.

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Algorithm : Proposed Q-NW-RBP
1: Initialize all  $m_{c \rightarrow v} = 0, m_{v \rightarrow c} = L_v^{ini}, \eta_{c_i} = 0$ 
2: Generate all  $m_{c \rightarrow v}^{pre}$ 
3: Set all  $r_{new}(c \rightarrow v) = m_{c \rightarrow v}^{pre} r_{old}(c \rightarrow v) = 0$ 
4: Compute all residual  $r(c \rightarrow v) = r_{new}(c \rightarrow v) - r_{old}(c \rightarrow v)$ 
5: Find  $r_{select} \rightarrow v_{select} = \alpha$ -th largest  $\{r(c_i \rightarrow v_j) | i \in \{1 \dots M\}, v_j \in \mathcal{N}(c_i), \eta_{c_i} < \lambda\}$ 
6: Set all  $r_{old} = r_{new}$ 
7: for every  $v_k \in \mathcal{N}(c_{select})$  do
  Generate and propagate  $m_{c_{select} \rightarrow v_k}$ 
  Set  $r_{(c_{max} \rightarrow v_k)} = 0$ 
  for every  $c_b \in \mathcal{N}(v_k) \setminus c_{select}$  do
    Generate and propagate  $m_{v_k \rightarrow c_b}$ 
  end for
  for every  $c_b \in \mathcal{N}(v_k) \setminus c_{select}$  do
    for every  $v_a \in \mathcal{N}(c_b) \setminus v_k$  do
      Generate  $r_{new}(c_b \rightarrow v_a) = m_{v_k \rightarrow c_b}^{pre}$ 
    end for
  end for
  for every  $c_b \in \mathcal{N}(v_k) \setminus c_{select}$  do
    for every  $v_a \in \mathcal{N}(c_b) \setminus v_k$  do
      Compute all residual  $r(c_b \rightarrow v_a)$ 
    end for
  end for
  Set all  $r_{old} = r_{new}$ 
end for
8:  $\eta_{c_{select}} = \eta_{c_{select}} + 1$ 
9: if all the CNs are satisfied or the maximum iteration number is achieved
  Break
else
  Return to 5.
10: Return the decoded results in the last decoder
    
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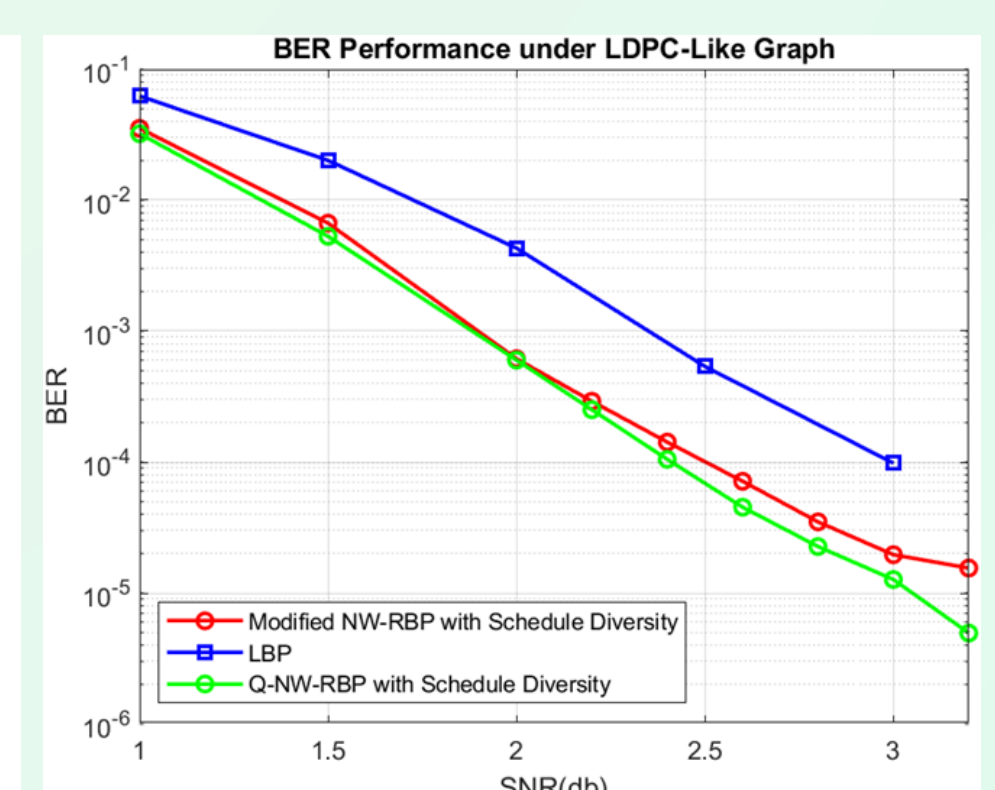


Fig. 3 (left) Algorithm of Q-NW-RBP
Fig.4 (right) performance of Q-NW-RBP with Schedule Diversity compare to other algorithm

Conclusion

The study show that in the first stage, combining LBP and Modified NW-RBP with schedule diversity significantly improves error performance compared with only LBP decoding. In high-SNR regions, it approaches the performance of conventional SCL decoding while requiring far fewer computations and much lower memory. In the second stage, incorporating Q-NW-RBP preserves performance at low SNR and further improves high-SNR behavior by mitigating trapping-set errors and reducing the error floor.